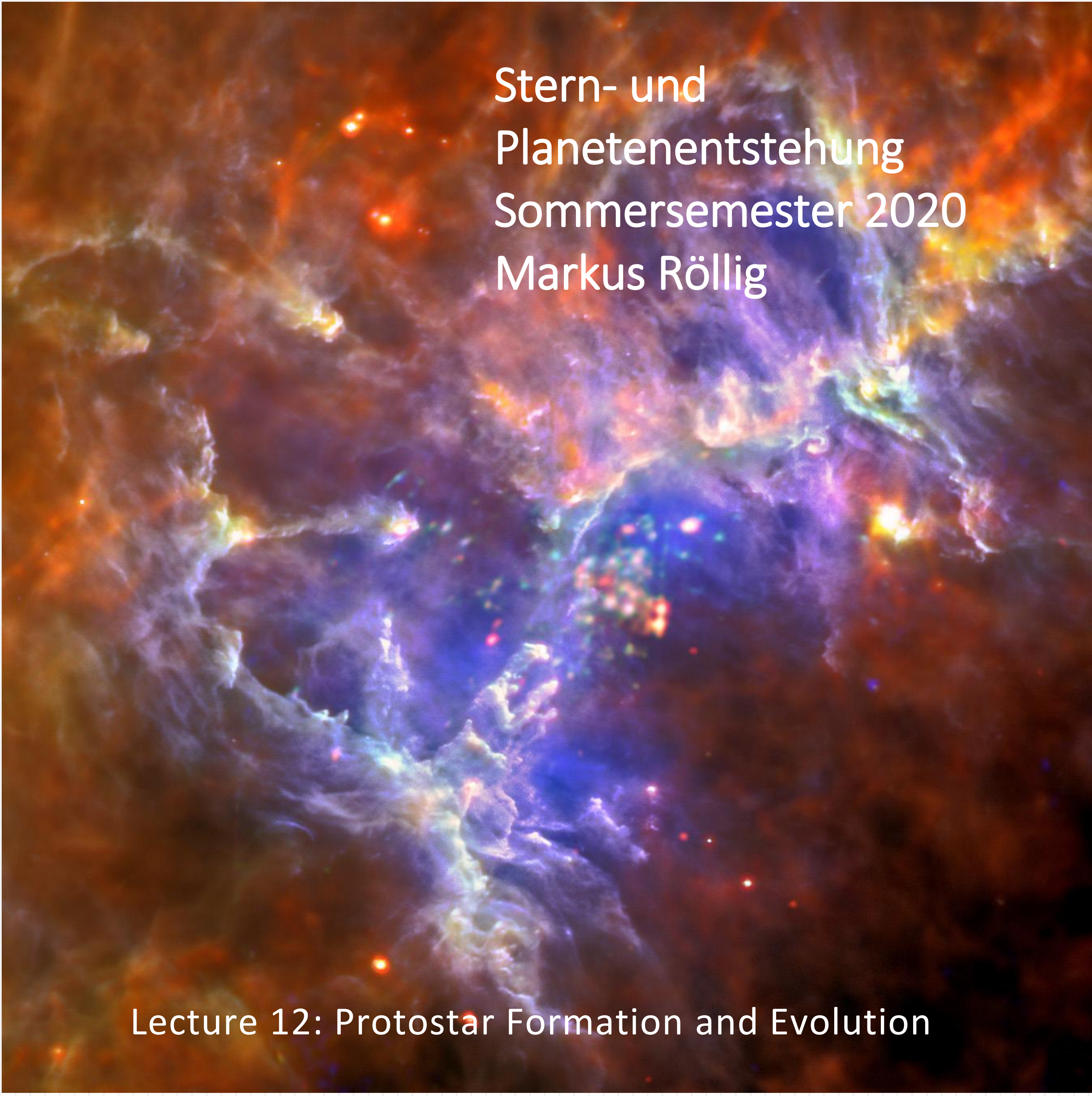


Stern- und Planetenentstehung
Sommersemester 2020

Markus Röllig

Lecture 12: Protostar Formation and Evolution

http://exp-astro.physik.uni-frankfurt.de/star_formation/index.php

VORLESUNG/LECTURE
Raum: Physik - 02.201a
dienstags, 12:00 - 14:00 Uhr

SPRECHSTUNDE:
Raum: GSC, 1/34, Tel.: 47433, (roellig@ph1.uni-koeln.de)
dienstags: 14:00-16:00 Uhr

Nr.	Thema	Termin
1	Observing the cold ISM	21.04.2020
2	Observing Young Stars	28.04.2020
3	Gas Flows and Turbulence Magnetic Fields and Magnetized Turbulence	05.05.2020
4	Gravitational Instability and Collapse	12.05.2020
5	Stellar Feedback	19.05.2020
6	Giant Molecular Clouds	26.05.2020
7	Star Formation Rate at Galactic Scales	02.06.2020
8	Stellar Clustering	09.06.2020
9	Initial Mass Function – Observations and Theory	16.06.2020
10	Massive Star Formation	23.06.2020
11	Protostellar disks and outflows – observations and theory	30.06.2020
12	Protostar Formation and Evolution	07.07.2020
13	Late Stage stars and disks – planet formation	14.07.2020

12 PROTOSTAR FORMATION AND EVOLUTION

Our goal will be to understand when and why collapse stops to form a pressure-supported object, and how those objects subsequently evolve into main sequence stars.

12.1 THERMODYNAMICS OF A COLLAPSING CORE

12.1.1 The Isothermal-Adiabatic Transition

The assumption of isothermality for cores that we have been making must break down at some point.

Polytrope $P \propto \rho^{\gamma}$ with $\gamma = 4/3$ the dividing line between EOS that are and are not stable against collapse ($\gamma < 4/3$: unstable).

Larson EOS: $T \propto \rho^{0.07}$, and $P \propto \rho^{1.07}$, therefore not stable against collapse.

In contrast, if the gas is not able to radiate at all, it will behave adiabatically. This means it will approach a polytrope with $\gamma = 7/5$ or $5/3$, depending on internal excitation conditions. (both > 4/3 and able to stop collapse)

Assuming a low angular momentum and a spherical collapse:

$$\frac{de}{dt} = \Gamma - \Lambda$$

(e: thermal energy per unit mass, Γ: heating, Λ: cooling)

Dominant heating term before central star forms is adiabatic compression:

$$\Gamma = -p \frac{d}{dt}\left(\frac{1}{\rho}\right)$$

$1/\rho$ is the specific volume, this term is just $p\, dV$. Compression timescale is about the free-fall timescale $t_{ff} \sim 1/\sqrt{G\rho}$, so we expect

$$\frac{d}{dt}\left(\frac{1}{\rho}\right) = C_1 \sqrt{\frac{4\pi G}{\rho}}$$

With some constant $C_1 \sim unity$. The factor 4π is inserted for later convenience. With $p = \rho c_s^2$ we get

$$\Gamma = C_1 c_s^2 \sqrt{4\pi G\rho}$$

Main cooling by dust emission.

1) Optical thin case:

$$\Lambda_{thin} = 4\kappa_p \sigma_B\, T^4$$

where σ_B is the Stefan-Boltzmann constant and κ_P is the Planck mean opacity of the dust grains. As long as $\Lambda \gtrsim \Gamma$ the gas can remain isothermal.

From $\Lambda = \Gamma$ we find

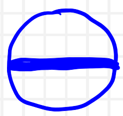
$$\rho_{thin} = \frac{4}{\pi} \frac{\kappa_P^2 \sigma_B^2 \mu^2 T^4}{C_1^2 G\, k_B^2}$$

$$= 5 \times 10^{-15} g\ cm^{-3} C_1^{-2} \left(100 \frac{\kappa_P}{cm^2 g^{-1}}\right)^2 \left(\frac{T}{10K}\right)^6$$

Thus, we find that compressional heating and optically thin cooling balance at about $10^{-14} g\ cm^{-3}$.

A second important density is the one at which the gas starts to become optically thick to its own re-emitted infrared radiation. Suppose that the optically thick region at the center of our core has some mean density ρ and radius R. The condition that the optical depth across it be unity then reduces to

$\tau = \kappa \cdot N$



$$2\kappa_P \rho R \approx 1$$

Size must be comparable to Jeans length if it is no longer collapsing

$$R \sim \lambda_J = \sqrt{\frac{\pi c_s^2}{G\rho}} = \frac{C_2 2\pi c_s}{\sqrt{4\pi G\rho}}$$

(Masunaga et al. find based on numerical results that $C_2 \approx 0.75$). We derive the characteristic density at which the gas transitions from optically thin to optically thick

$$\rho_{\tau\sim 1} = \frac{1}{4\pi} C_2^{-2} \frac{\mu G}{\kappa_p^2 k_B T}$$

$$= 1.5 \times 10^{-13} g\ cm^{-3} C_2^{-2} \left(\frac{100\kappa_P}{cm^2 g^{-1}}\right)^{-2} \left(\frac{T}{10K}\right)^{-1}$$

Note that,

$$\frac{\rho_{thin}}{\rho_{\tau\sim1}} \propto \kappa_P^4\, T^7$$

So small changes in κ_P or T will change them dramatically! E.g. super-solar metallicity: κ_P will be larger → ρ_{thin} increases, $\rho_{\tau\sim1}$ decreases (same for higher temperatures).

If $\rho_{thin} < \rho_{\tau\sim1}$: collapsing gas will become opt. thick before heating becomes faster than opt. thin cooling – we need to compare with opt. thick cooling.

If $\tau \gg 1$, the effective speed of the radiation moving through it is c/τ, so the time required for the radiation to diffuse out is

$$t_{diff} = \frac{l\,\tau}{c} = \frac{\kappa_P \rho\, l^2}{c}$$

Inside the opt. thick region: thermal balance between radiation and matter, so radiation energy density is aT^4 . Radiation energy per unit mass=$\sigma_B T^4/\rho$ and $l = 2R$, we get (with $\sigma_B = ca/4$)

$$\Lambda_{thick} = \frac{aT^4/\rho}{t_{diff}} = \frac{\sigma_B T^4}{\kappa_P \rho^2 R^2}$$

If we equate Λ_{thick} and Γ, we get the characteristic density where the gas becomes non-isothermal in the optically thick regime

$$\rho_{thick} = \left(\frac{C_1^2 G \sigma_B^2 \mu^4 T^4}{4\pi^3 C_2^4 k_B^4 \kappa_P^2}\right)^{1/3}$$

$$= 5 \times 10^{-14} g\ cm^{-3} \frac{C_1^{\frac{2}{3}}}{C_2^{\frac{4}{3}}} \left(100 \frac{\kappa_P}{cm^2 g^{-1}}\right)^{-\frac{2}{3}} \left(\frac{T}{10K}\right)^{4/3}$$

C_1, C_2: of order unity

Less dependent on κ_p and T. In general, for reasonable collapse conditions we expect that cores transition from isothermal to close to adiabatic at a density of $\rho \sim 10^{-13} - 10^{-14}\ \mathrm{g\ cm^{-3}}$.

12.1.2 The First Core

The transition to an adiabatic equation of state, with $\gamma > 4/3$, means that the collapse must at least temporarily halt. The result will be a hydrostatic object that is supported by its own internal pressure. This object is known as the first core, or sometimes a Larson's first core.

We can model the first core reasonably well as a simple polytrope, with index n defined by $n = 1/(\gamma - 1)$. At low mass when the temperature in the first core is low $\gamma \approx 5/3$ and $n \approx 3/2$, and for a more massive, warmer core $\gamma \approx 7/5$ and $n \approx 5/2$. For $\gamma \approx 5/3$ we find:

$$R = 2.2\ \mathrm{AU} \left(\frac{10^{10}\rho_c}{\mathrm{g\ cm^{-3}}}\right)^{\frac{1}{6}} \left(\frac{T}{10\mathrm{K}}\right)^{\frac{1}{2}} \left(\frac{10^{13}\rho_{ad}}{\mathrm{g\ cm^{-3}}}\right)^{-\frac{1}{3}}$$

$$T = 0.059\ M_\odot \left(\frac{10^{10}\rho_c}{\mathrm{g\ cm^{-3}}}\right)^{\frac{7}{6}} \left(\frac{T}{10\mathrm{K}}\right)^{\frac{1}{2}} \left(\frac{10^{13}\rho_{ad}}{\mathrm{g\ cm^{-3}}}\right)^{-1/3}$$

ρ_{ad}: density at which the gas becomes adiabatic

For $\gamma \approx 7/5$ we obtain very similar numbers.

These results show that the first core is an object a few AU in size, with a mass of a few hundredths of a solar mass.

12.1.3 Second Collapse

The first core is a very short-lived phase in the evolution of the protostar. To see why, let us estimate its temperature. The temperature inside the sphere rises as $T \propto \rho^{\gamma-1}$ so the central temperature is

$$T_c = T_0 \left(\frac{\rho_c}{\rho_{ad}}\right)^{\gamma-1}$$

T_0: temperature in the isothermal phase.

Thus, the central temperature will be higher than the boundary temperature by a factor that is determined by how high the central density has risen, which in turn will be determined by the amount of mass that has accumulated on the core. With $M \propto \rho_c^{\frac{(3+n)}{2n}} = \rho_c^{(3\gamma-2)/2}$ and $T_c \propto \rho_c^{\gamma-1}$ we get

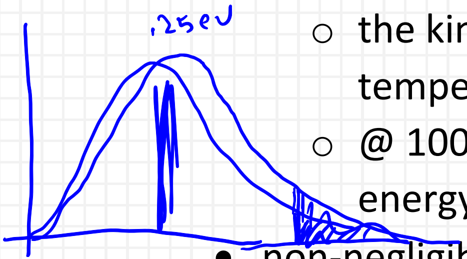
$$T_c \propto M^{(2\gamma-2)/(3\gamma-2)}$$

The exponent is 0.44 for $\gamma = 5/3$ and 0.36 for $\gamma = 7/5$.

Example: $M = 0.06\ \mathrm{M}_\odot, \rho_{ad} = 10^{-13}\mathrm{g\ cm}^{-3}, \gamma = \frac{5}{3}$ gives

$$\rho_c = 10^{-10}\mathrm{g\ cm}^{-3} \text{ and } T_c = 1000\ \mathrm{K}$$

- by the time ~ $0.1\ M_\odot$ of material has accumulated on the first core
  - compression will have caused its central temperature to rise to 1000 K or more.
- This causes yet another change in the thermodynamics of the gas
  - because all the hydrogen is still molecular
  - molecular hydrogen has a binding energy of 4.5 eV.
  - the kinetic energy per molecule for molecular hydrogen at a temperature T is $3k_BT = 0.26\ T_3$ eV.
  - @ 1000 K => mean molecule has only ~5% of the kinetic energy that would be required to dissociate it.



- non-negligible tail of the Maxwellian distribution that is moving fast enough for collisions to produce dissociation.
  - Each dissociative collision removes 4.5 eV from the kinetic energy budget of the gas and puts it into chemical energy instead.
- dissociations are occurring on the tail of the Maxwellian
  - any slight increase in the temperature dramatically increases the dissociation rate
  - moving even more kinetic energy into chemical energy.
- Thermostat for the gas.
  - Detailed numerical calculations of this effect show that at temperatures above 1000 - 2000 K, the equation of state becomes closer to $T \propto \rho^{0.1}$, or $\gamma = 1.1$.
  - This is again below the critical value

    - center of the first core again goes into something like free-fall collapse.
- This is called the second collapse.
- This collapse continues until all the hydrogen dissociates.
    - The hydrogen also ionizes during this collapse,
    - ionization potential of 13.6 eV isn't very different from the dissociation potential of 4.5 eV.
    - Only once all the hydrogen is dissociated and ionized can a new hydrostatic object form.
- At this point the gas is warmer than ~10^4 K, is fully ionized, and the new hydrostatic object is a true protostar.
    - This implies that Brown Dwarfs do not undergo a prompt second collapse.
    - First core never massive enough to dissociate H2 in its center.
    - Second collapse only possible after long and slow energy loss by radiation from surface.

12.2 THE PROTOSTELLAR ENVELOPE

Once a protostar is born at the center of a collapsing cloud, we can ask both about the structure immediately around it and about its internal structure.

12.2.1 Accretion Luminosity

The central driver of the behavior of the gas around the protostar is the radiation that it emits.

- Early: star has not reached MS or nuclear burning phase
- gravity is main energy source
- gas striking the hydrostatic protostar comes to a halt in an accretion shock
- kinetic energy radiated away

$$L_{acc} = \frac{GM_\star \dot{M}_\star}{R_\star}$$

$R_\star$: radius of protostar (~ few $R_\odot$), $\dot{M}_\star \sim 10^{-5} M_\odot yr^{-1}$

$$L_{acc} = 30 L_\odot \dot{M}_{\star,-5} M_{\star,0} R_{\star,1}^{-1}$$

$\dot{M}_{\star,-5} = \dot{M}_\star/(10^{-5} M_\odot yr^{-1})$, $M_{\star,0} = M_\star/M_\odot$, $R_{\star,1} = R_\star/(10R_\odot)$

typical low-mass star L_{acc} ~ many tens of $L_\odot$ (much more than from nuclear burning on MS)

The effective temperature of the stellar surface due to accretion:

$$T_2 = 1.2 \times 10^6 M_{\star,0} R_{\star,1}^{-1}\ \mathrm{K}$$

- ➔ The post-shock gas is heated to temperatures such that it emits in UV and x-rays.
- ➔ incoming gas optically thick to that radiation
- ➔ radiation emitted by the post-shock gas will be absorbed in a small region immediately outside the shock
- ➔ reprocessed until it becomes blackbody emission

$$L_{acc} = 4\pi R_\star^2 T_\star^4$$
$$T_\star = 4300\ \dot{M}_{\star,-5}^{1/4} M_{\star,0}^{1/4} R_{\star,1}^{-3/4}\mathrm{K}$$

Surface temperature comparable to MS star.

12.2.2 Dust Destruction Front

Consider: spherical, black dust grain of radius a some distance r from the star. It absorbs radiation at a rate:

$$\Gamma = \frac{L_{acc}}{4\pi r^2}\pi a^2 = \pi a^2 \sigma\, T_\star^4 \left(\frac{R_\star}{r}\right)^2$$

and radiates it at a rate:

$$\Lambda = 4\pi a^2 \sigma_3 T_d^4$$

Therefore:

$$\boxed{T_d = \left(\frac{R_\star}{2r}\right)^{1/2} T_\star}$$

- graphite and silicate grains, will vaporize at temperatures larger than ~1000-1500 K.
- when $r/R_\star$ is too large grains cannot survive
- we expect the proto-star to be surrounded by a dust-free region
- Gas in this region is mostly neutral and transparent to the proto-star radiation

- opacity gap
- dust destruction radius is the minimum radius where dust grains can exist

$$r_d = \frac{R_\star}{r}\left(\frac{T_\star}{T_d}\right)^2 = 0.4\, T_{d,3}^{-2} \dot{M}_{\star,-5}^{1/2} M_{\star,0}^{1/2} R_{\star,1}^{-1/2}\ \mathrm{AU}$$

Thus, the dust free regions extend to ~1 AU around an accreting proto-star.

12.2.3 Temperature Structure and Observable Properties

Now let us consider the material beyond the dust destruction front. At the front the gas density is given roughly by the condition

$$\dot{M}_\star = 4\pi r_d^2 \rho v_{ff}$$

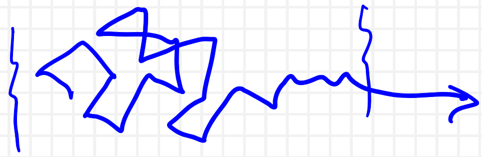
$$\rho = \frac{\dot{M}_\star}{\sqrt{8\pi^2 G M r_d^3}}$$

$$= 4\times 10^{-13} \dot{M}_{\star,-5}^{1/4} M_{\star,0}^{-7/4} R_{\star,1}^{3/4} T_{d,3}^{3}\ \mathrm{g\ cm^{-3}}$$

- Inside the front, the spectrum is a blackbody of a few 1000 K, so the peak wavelength is

$$\lambda \approx \frac{hc}{4kT} = 440 \dot{M}_{\star,-5}^{-\frac{1}{4}} M_{\star,0}^{-\frac{1}{4}} R_{\star,1}^{\frac{3}{4}}\ \mathrm{nm}$$

- opacity @440nm: $\kappa = 8000\ \mathrm{cm^2 g^{-1}}$, so the mean free path of stellar photons is $(\kappa\rho)^{-1} \approx 3\times 10^8 \mathrm{cm}$ (this is tiny!)
- Thus all the starlight that strikes the dust destruction front will immediately be absorbed by the dust grains.
- They will re-emit it as thermal radiation with a peak wavelength determined by their blackbody temperature (factor ~ 4 lower than the stellar surface)
- @ 1.8µm (~4x440nm) the opacity is $1000\ \mathrm{cm^2 g^{-1}}$, so the mean free path is a factor 8 higher.
- This is still tiny, so all the re-emitted radiation will also be absorbed.
- Radiation must diffuse out through the dust envelope.



$$F = -\frac{c}{3\rho\kappa_R}\nabla E$$

F: radiation flux, E: radiation energy density
κ_R: Rosseland mean opacity

- The repeated absorption and re-emission of radiation forces it into thermal equilibrium with the gas, so E is simple the energy density of a thermal radiation field at the gas temperature $E = \sigma T^4$
- No energy is added or removed from the radiation filed: $F = L_{acc}/(4\pi r^2)$

$$L_{acc} = -\frac{16\pi c\sigma r^2}{3\rho\kappa_R} T^3 \frac{dT}{dr}$$

For a given density structure and a model of dust grains that specifies $\kappa_R(T)$, this equation allows us to estimate the temperature structure in the proto-stellar envelope.

For reasonable grain model: $\kappa_R \propto T^\alpha$ with $\alpha \approx 0.8$

We assume $\rho \propto r^{-k_\rho}$ and $T \propto r^{-k_T}$

$$k_T = \frac{k_\rho + 1}{4 - \alpha}$$

In the free-falling part of the envelope $k_\rho \approx 3/2$, and $k_T \approx 0.8$.

In our fiducial example, where the temperature is 1000 K at 0.4 AU, we would expect the temperature to drop to 300 K at around 2 AU, to 100 K at around 8 AU, and back to the background temperature of 10 K at around 150 AU.

Once the dust temperature falls off to less than ~100 K, depending on the size of the core, the radiation is free to escape instead.

More detailed structure determination needs numerical models.

12.3 Proto-stellar Evolution

Extend the theory of stellar structure to the case of proto-stars that are not yet on the main sequence. There are only two significant differences.

- First, the boundary conditions are obviously different, since proto-stars are gaining mass from the outside.
- Second, although the star is in hydrostatic balance, it need not be in long-term thermal balance

12.3.1 Fundamental Theory

12.3.1.1 Time Scales

Let us begin by justifying the statement that we can treat proto-stars using the same techniques we use for main sequence stars. The way we can check this is by evaluating two time scales: the mechanical time scale over which the star will reach mechanical equilibrium, and the Kelvin-Helmholtz time scale over which it reaches thermal equilibrium.

The time to reach mechanical equilibrium (sound crossing time):

$$t_s \approx \frac{R}{c_s} = \sqrt{\frac{R^3}{GM}} = 35 M_0^{-\frac{1}{2}} R_1^{\frac{3}{2}} \text{ hours} \qquad M_0 = M_\star/M_\odot \quad R_{\star,1} = R/(10R_\odot)$$

In contrast, the time required to reach thermal equilibrium is the Kelvin-Helmholtz time, which is defined as roughly the time required for the star to radiate away its own binding energy:

$$t_{KH} = \frac{GM^2}{RL} = 3 \times 10^5 M_0^2 R_1^{-1} L_1^{-1} \text{ yr}$$

- mechanical equilibrium reached instantaneous compared to thermal equilibrium
- reasonable to assume that at all times the star is in hydrostatic balance
- accretion time ($\sim 1 M_\odot$ star in 10^5 yr) is shorter than KH time
- stars will cease accreting before they reach thermal equilibrium (only true for low mass stars)

12.3.1.2 Evolution Equations

Calculations are most convenient to work in Lagrangian coordinates, where we let M_r be the mass interior to radius *r*.

The first equation is the standard definition of mass in terms of density and radius

$$\frac{\partial r}{\partial M_r} = \frac{1}{4\pi r^2 \rho}$$

The second equation is the equation of hydrostatic balance.

$$\frac{\partial P}{\partial M_r} = -\frac{GM_r}{4\pi r^4}$$

The third equation is the equation of radiation diffusion (applies only as long as the proto-star is stable against convection):

$$T^3 \frac{\partial T}{\partial M_r} = -\frac{3\kappa L}{256\pi^2 \sigma r^4}$$

The last equation describes how the internal energy of the fluid evolves

$$\frac{\partial L}{\partial M_r} = \epsilon - T\frac{\partial s}{\partial t}$$

where s is the entropy per unit mass of the gas and ϵ is the rate of nuclear energy generation per unit mass.

This is the only equation that is different from the case of a main sequence star. For a main sequence star, we simply assume that the entropy per unit mass is constant, so we drop the $\partial s/\partial t$ term.

We require some additional equations:

The equation of state:

$$P = \frac{\rho k_B T}{\mu}$$

The entropy for a constant μ:

$$s = \frac{k}{\mu}\ln\left(\frac{T^{\frac{3}{2}}}{\rho}\right) + \text{const}$$

12.3.1.3 Boundary conditions

The four structure equations require four boundary conditions to solve. Two are obvious; at $M_r = 0$

$$r(0) = 0$$
$$L(0) = 0$$

The remaining two boundary conditions, describing the pressure and luminosity at the edge of the star, are different from a MS star.

$$P(M_\star) = \frac{\dot{M}}{4\pi}\sqrt{\frac{2GM_\star}{R_\star^5}}$$

The final boundary condition is on the luminosity. For an accreting star it is more complicated, however, because the accreting gas carries energy, and the question becomes what fraction of this energy will be radiated away at the stellar surface and what fraction will be advected or radiated into the stellar interior.

$$L(M_\star) = L_{acc} + L_{bb} + L_{in}$$

$L_{acc} = GM_\star \dot{M}_\star / R_\star$, is the kinetic energy of the accreting gas,

$L_{bb} = 4\pi R_\star^2 \sigma T(M_\star)^4$, is the blackbody radiation from the stellar surface

L_{in}: represents the inward flux of energy due to advection and radiation from the shocked gas.

Let's briefly look at the nuclear energy generation:

12.3.1.4 Deuterium Burning

- For a MS star, ϵ comes from fusion of hydrogen into helium (pp chain or CNO cycle)
- hydrogen burning doesn't occur just before the star reaches the MS
- another nuclear reaction that can occur at lower temperatures: fusion of deuterium

$$^2\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma$$

- This reaction begins to occur at an appreciable rate once the temperature reaches 10^6 K
- the reaction releases 5.5 MeV per deuterium nucleus burned.

$$\epsilon \approx \begin{cases} 0, & T < 10^6\ \mathrm{K} \\ 4.19 \times 10^7 [\mathrm{D/H}]\, \rho_0 T_6^{11.8} \mathrm{erg\, g^{-1} s^{-1}}, & T > 10^6\ \mathrm{K} \end{cases}$$

$[\mathrm{D/H}] \approx 2 \times 10\text{^} - 5$: D to H ratio in the gas, $\rho_0 = \rho/(1\mathrm{g\,cm^{-3}})$

$T_6 = T/(10^6\mathrm{K})$

12.3.1.5 *Numerical Solution*

To construct a numerical model, we need to specify the accretion rate $\dot{M}_\star$.

We must also start with an initial condition, which we usually take to be a simple polytrope. This gives us initial profiles of *r, P, T*, and *L*, from which we can obtain other derived variables like ρ and *s*, as a function of M_r.

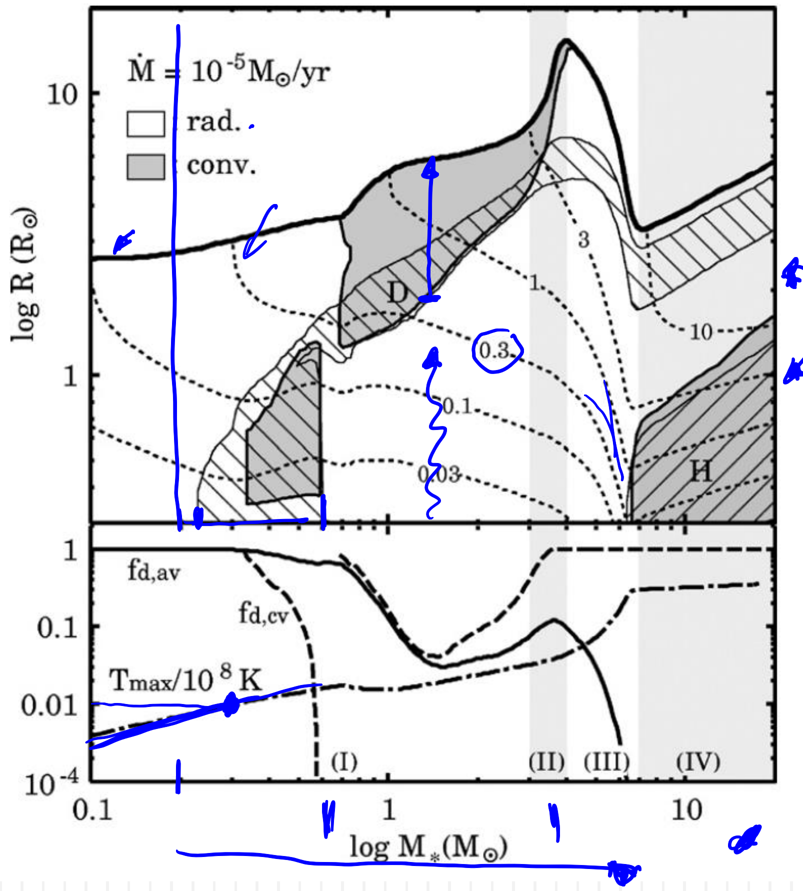


Figure 18.1: Kippenhahn and composition diagrams for a protostar accreting at $10^{-5} M_\odot$ yr^{-1} (Hosokawa & Omukai, 2009). In the top panel, the thick curve shows the protostellar radius as a function of mass, and gray and white bands show convective and radiative regions, respectively. Hatched areas show regions of D and H burning, as indicated. Thin dotted lines show the radii containing 0.1, 0.3, 1, 3, and 10 $M_\odot$, as indicated. Shaded regions show four evolutionary phases: (I) convection, (II) swelling, (III) KH-contraction, and (IV) the main sequence. In the lower panel, the solid line shows the mean deuterium fraction in the star, normalized to the starting value, while the dashed line shows the D fraction only considering the convective parts of the star. The dot-dashed line shows the maximum temperature.

12.3.2 Evolutionary Phases for Proto-stars

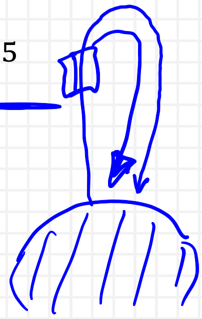
We will use as our primary example the case of a star undergoing hot accretion at $10^5 M_\odot \mathrm{yr}^{-1}$, as illustrated in the Figure above.

- The initial phase of evolution is visible as what takes place up to a mass of $\approx 0.2\ M_\odot$
  - star reaches a radius that is solely a fact of $M_\star$ and $\dot{M}_\star$
  - radius determined by entropy profile
    - high entropy leads to high radius (since internal energy generated by the star is small compared to the accretion power when the stellar mass is low)

        - Higher accretion rates bury accreted material more quickly, leaving it with higher entropy and producing larger radii.
    - Regardless of the boundary condition assumed, during this phase there is no nuclear burning in the star, as the interior is too cold for any such activity.
- The next evolutionary phase begins at $\approx 0.25\ M_\odot$, and continues to $\approx 0.7\ M_\odot$
    - onset of nuclear burning
    - onset of convection
    - The driving force behind both phenomena is that, as the proto-star gains mass, its interior temperature rises $\propto T^{11.8}$
    - The energy generation rate is incredibly sensitive to T
        - any slight raise in the temperature causes it to jump enough to
        - raise the pressure and
        - adiabatically expand the star
        - reducing T.
        - T is fixed to ~10^6K
    - The star adjusts its radius accordingly to keep the center temperature constant
        - deuterium burning halts core contraction temporarily (apparent from the way the dotted lines showing constant mass enclosed bend upward at $\approx 0.3\ M_\odot$)
        - deuterium burning that it causes a rapid rise in the entropy at the center of the star
            - This has the effect of starting up convection in the star.
        - nearly-linear mass-radius relation
- The next evolutionary phase begins at $\approx 0.6\ M_\odot$, and continues to $\approx 3\ M_\odot$
    - exhaustion of deuterium in the stellar core
        - each deuterium burned provides 5.5 MeV compared to 7MeV per hydrogen provided by the hydrogen burning.
        - 2×10^{-5} D nuclei per H nuclei
    - time required for a star to exhaust its deuterium is

$$t_{\mathrm{D}} = \frac{[\mathrm{D/H}]\Delta E_{\mathrm{D}} M_\star}{m_{\mathrm{H}} L_\star} = 1.5 \times 10^6 \mathrm{yr}\, M_{\star,0} L_{\star,0}^{-1}$$

- deuterium burning will hold up a star's contraction briefly.
- t_D still comparable to formation time
  - stars may burn deuterium for most of the time they are accreting
- deuterium supply begins to run out,
- the rate of energy generation in the core becomes insufficient to prevent it from undergoing further contraction
- leading to rising temperatures.
- rise in central temperature lowers the opacity $\kappa \propto \rho T^{-3.5}$
  - easier energy transport outward
  - shuts off convection eventually (radiative barrier)
  - ends D transport to the core
- core can resume contraction



- D continues to burn in the shell

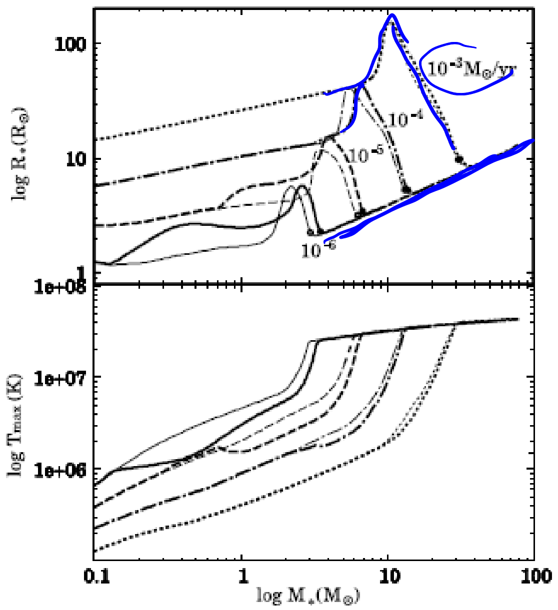


Figure 18.2: Radius versus mass (top panel) and maximum temperature versus mass (bottom panel) for protostars accreting at different rates. The accretion rate is indicated by the line style, as illustrated in the top panel. For each accretion rate there are two lines, one thick and one thin. The thick line is for the observed Milky Way deuterium abundance, while the thin line is the result assuming zero deuterium abundance.

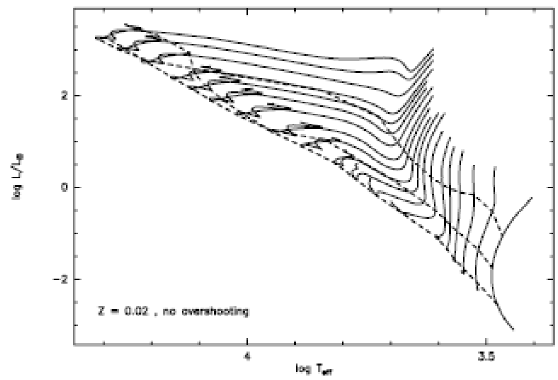


Figure 18.3: Solid lines show tracks taken by stars of varying masses, from 0.1 $M_\odot$ (rightmost line) to 7.0 $M_\odot$ (leftmost line) in the theoretical HR diagram of luminosity versus effective temperature. Stars begin at the upper right of the tracks and evolve to the lower left; tracks end at the main sequence. Dashed lines represent isochrones corresponding to 10^6, 10^7, and 10^8 yr, from top right to bottom left. Figure from Siess et al. (2000).

- The next evolutionary phase occurs from $\approx 3 - 4\ M_\odot$ and is called swelling
  - marked increase in the star's radius over a relative short period of time
  - The physical mechanism driving this is the radiative barrier discussed above
  - The decreased opacity allows the center of the star to lose entropy rapidly, and the entropy to be transported to the outer parts of the star via radiation.

    - The result is a wave of luminosity and entropy that propagates outward through the star
    - Once the wave of luminosity and entropy gets near the stellar surface, which is not confined by the weight of overlying material, the surface undergoes a rapid expansion, leading to rapid swelling.
- The final stage is the contraction to the main sequence
    - Entropy wave hits surface
        - star is able to lose energy and entropy quickly
        - resumes contraction
        - contraction ends once core temperature is sufficient for hydrogen burning.

12.3.3 Observable Evolution of Proto-stars

12.3.3.1 *The Birthline*

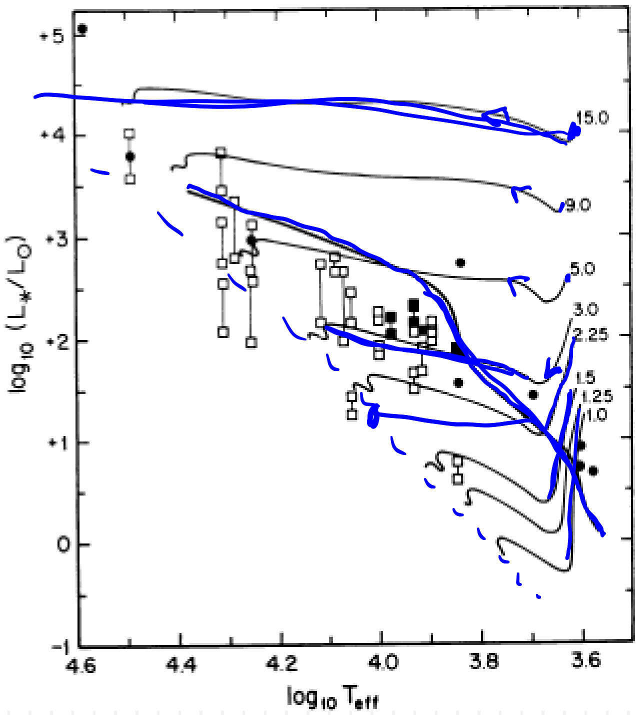


Figure 18.4: Thin lines show tracks taken by stars of varying masses (indicated by the annotation, in $M_\odot$) in the theoretical HR diagram of luminosity versus effective temperature. Stars begin at the upper right of the tracks and evolve to the lower left; tracks end at the main sequence. The thick line crossing the tracks is the birthline, the point at which the stars stop accreting and become optically visible. Squares and circles represent the properties of observed young stars. Figure taken from Palla & Stahler (1990).

- As long as a star is accreting it will not be visible in the optical.
- Stars appear in the HR diagram only after they finished their main accretion phase.

- Stars still accreting: **proto-stars**
- Stars in post-accretion phase: **pre-main sequence stars**
- For stars below $\sim 1 M_{\odot}$ the transition from proto-star to pre-main sequence star will occur some time after the onset of deuterium burning, either during the core or shell burning phases depending on the mass and accretion history.
- More massive stars will become visible only during KH contraction, or even after the onset of hydrogen burning.
- Since there is a strict mass-radius relation during core deuterium burning (with some variation due to varying accretion rates), there must be a corresponding relationship between L and T, just like the main sequence
- We call this line in the HR diagram, on which proto-stars first appear, the birthline
  - young stars are larger and more luminous than MS stars
  - birthline at higher L and lower T than MS stars

12.3.4 The Hayashi Track

- For low mass stars the initial phases of evolution in the HR diagram are nearly vertical, i.e. constant T_{eff}
- The vertical tracks for different masses are very close together
- This vertical part of the evolution is called the Hayashi track
  - origin in opacity at low temperatures
  - at below 10^4K, hydrogen become neutral
  - free electrons come only from metals
  - some form H^- , the dominant source of opacity
  - opacity will be so low that the optical depth to infinity will be < 2/3
    - no photosphere ($\tau = 1$)
  - there is a minimum surface temperature to maintain a $\tau = 1$ surface
    - $T_{min} = T_H = 3500\mathrm{K}$

12.3.5 The Henyey Track

- Contraction at nearly constant T_{eff} continues until the star contracts enough to raise its surface temperature above T_H
- Increase in T
    - star transition from convective to radiative
- In the HR diagram, contraction and increase in T_{eff} produces a ~ horizontal evolutionary track
    - This is called the Henyey track
    - star continues to contract until center becomes warm enough for H burning
- star is now on MS